LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.C.A. DEGREE EXAMINATION – COMPUTER APPLICATION

FIRST SEMESTER – NOVEMBER 2007

MT 1902 /CA 1804- MATHEMATICS FOR COMPUTER APPLICATIONS

AL 2

Date : 02/11/2007 Dept. No. Max. : 100 Marks

Time : 1:00 - 4:00

SECTION A

Answer ALL the questions. (10 x 2 = 20)

1. Define a Boolean algebra.
2. What are the three connectives used in the object language?
3. Construct the truth table for $P \vee \urcorner P$.
4. Define Phrase structure grammar.
5. Construct a regular grammar for the language $L = \{a^n b^m / n, m \geq 1\}$
6. Define a Non – Deterministic finite automata.
7. Which of the following relations in the set of human beings are equivalence relations (i) $R = \{(a,b) : a \text{ is wife of b}\}$ (ii) $R = \{(a,b) : a \text{ is brother of b}\}$.
8. Show that if any five integers from 1 to 8 are chosen, then at least two of them will have a sum 9.
9. State Kuratowski's theorem for planarity.
10. If $a \in G$ and $a^n = e$, prove that $o(a)$ divides n.

SECTION B

Answer ALL the questions. (5 x 8 = 40)

11. (a) Prove that a bijective map of a lattice L into a lattice L' is a lattice isomorphism if and only if its inverse is order preserving.

(or)

(b) Prove that the complement a' of any element 'a' of a Boolean algebra B is uniquely determined. Prove also that the map $a \to a'$ is an anti-automorphism of period ≤ 2 and $a \to a'$ satisfies $(a \vee b)' = a' \wedge b'$, $(a \wedge b)' = a' \vee b'$, $a'' = a$.

12. (a) Discuss about Negation, Conjuction and Disjunction connectives.

(or)

(b) Consider $G = (V, T, P, S)$, where $V = \{S, A, B\}, T = \{a, b\}$, and P consists of the following:

$S \to aB$ $S \to bA$

$A \to a$ $A \to aS$ $A \to bAA$

$B \to b$ $B \to bS$ $B \to aBB$

Prove that the language $L(G)$ is the set of all words T^+ consisting of an equal number of a's and b's.

13. (a) (i) Define Context – free grammar.

(ii) Construct a context sensitive grammar for the language $L = \{a^n b^m a^n / n, m \geq 1\}$.

(or)

(b) Let L be a set accepted by a non-deterministic finite automaton. Then prove that there exists a deterministic finite automaton that accepts L.

14. (a) Give an example of a relation which is:

(i) reflexive and transitive but not symmetric

(ii) symmetric and transitive but not reflexive

(iii) reflexive and symmetric but not transitive

(or)

(b) (i) Show that the relation R = $\{(a, b) / a - b = k\,m$ for some fixed integer m and $a, b, k \in Z\}$ is an equivalence relation.

(ii) A man has 7 relatives, 4 of them are ladies and 3 gentlemen and his wife has 7 relatives, 3 of them are ladies and 4 gentlemen. In how many ways can they invite ladies and 3 gentlemen for a dinner party so that there are 3 of man's relatives and 3 of his wife's relatives?

15. (a) (i) Prove that there is a one-to-one correspondence between any two left cosets of a subgroup H in G.
(ii) Prove that a subgroup N of a group G is a normal subgroup of G iff every left coset of N in G is a right coset of N in G.

(or)

(b) (i) If G is a graph in which the degree of every vertex is at least two then prove that G contains a cycle.
(ii) Prove that a closed walk of odd length contains a cycle.

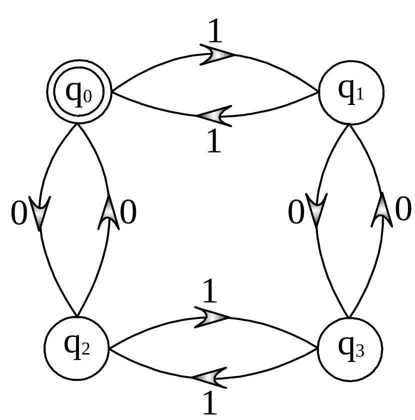
SECTION C

Answer any TWO questions. **(2 x 20 = 40)**

16. (a) Prove that a non-empty set L together with two binary operations $\wedge$ and $\vee$ is said to form a lattice if and only if for every $a, b, c \in L$, the following conditions hold.

L_1: $a \wedge a = a$, $a \vee a = a$.
L_2: $a \wedge b = b \wedge a$, $a \vee b = b \vee a$.
L_3: $a \wedge (b \wedge c) = (a \wedge b) \wedge c$, $a \vee (b \vee c) = (a \vee b) \vee c$.
L_4: $a \wedge (a \vee b) = a$, $a \vee (a \wedge b) = a$.

(b) For the finite automaton $M = (Q, \Sigma, \delta, q_0, F)$,



give the transition table and show that 11010010 is in $L(M)$.

(15 + 5)

17. (a) Write a short note on principal disjunctive normal form and construct an equivalent formula for $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$.
(b) State and prove the pumping lemma for regular sets.

(10 + 10)

18. (a) Let G be a (p, q) graph. Then prove that the following statements are equivalent
(i) G is a tree.
(ii) Every two points of G are joined by a unique path.
(iii) G is connected and p = q + 1.
(iv) G is acyclic and p = q + 1.
(b) Show that the intersection of two normal subgroups of G is a normal subgroup of G.

(14 + 6)
